Q-1(5)

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Equation of a Progressive wave [contd....]

Hence the displacement at P at any instant t can be obtained by substituting $\left(t-\frac{x}{v}\right)$ in place of t in the above eqn.(1)

$$\therefore \quad y = a \sin \omega \left(t - \frac{x}{v}\right) \qquad — \quad (2)$$

$$\because \quad \omega = 2\pi n$$

$$\therefore \quad y = a \sin 2\pi n \left(t - \frac{x}{v}\right) \qquad — \quad (3)$$

Again $\quad \omega = \frac{2\pi}{T}$

$$\therefore \quad y = a \sin 2\pi \left(t - \frac{x}{vT}\right)$$

The period T is the time required for the wave to travel a distance of one wavelength λ, so that $vT = \lambda$. Thus

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad — \quad (4)$$

But this may also be written as

$$y = a \sin \frac{2\pi}{\lambda} \left(\frac{\lambda t}{T} - x\right) \qquad —$$

But $\lambda/T = v$

$$\therefore y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——(5)}$$

also $\frac{2\pi}{\lambda} = k$ is called propagation const.

Hence the above eqn. becomes

$$y = a \sin k(vt - x) \quad \text{—— (6)}$$

$$\because \quad v = n\lambda = \frac{\omega}{2\pi} \cdot \lambda = \frac{\omega}{2\pi/\lambda} = \frac{\omega}{k}$$

$$\therefore \quad y = a \sin k\left(\frac{\omega}{k} \cdot t - x\right)$$

$$\therefore \quad y = a \sin (\omega t - kx) \quad \text{——(7)}$$

Eqn (2), (3), (4), (5), (6) & (7) represent the different forms of eqn. of plane progressive wave travelling along +X-direction, where it has been assumed that the displacement y is zero at the point $x = 0$ and $t = 0$. The general expression for the wave motion may be written as

$$y = a \sin(\omega t - kx + \phi)$$

where ϕ is called phase constant which represents the phase difference between the above wave travelling along +X direction and another wave.